Tutorial 2

Week of September 17, 2018

**Review summary of transformations given at the bottom of page 37 of your textbook.

1. Sketch the following:
   (a) $y = 2\sqrt{x+1}$
      i. Begin by graphing $y = \sqrt{x}$.
      ii. Shifting our graph over to the left one unit gives $y = \sqrt{x+1}$.
      iii. Multiplying all our y values by 2, we obtain $y = 2\sqrt{x+1}$.
   (b) $y = 2 - \sqrt{x}$
      i. Begin by graphing $y = \sqrt{x}$.
      ii. Flipping the graph across the x-axis (i.e. positive y-values become negative) gives us $y = -\sqrt{x}$.
      iii. We can rewrite $2-\sqrt{x}$ as $-\sqrt{x}+2$ meaning that we need to move our graph 2 units upwards, giving us $y = 2 - \sqrt{x}$.
   (c) $y = |\sqrt{x} - 1|$
      i. Begin by graphing $y = \sqrt{x}$.
      ii. Moving down 1 unit gives $y = \sqrt{x} - 1$.
      iii. We apply the absolute value to our function which makes all our negative y values positive, giving us $y = |\sqrt{x} - 1|$.
   (d) $y = 3 - 2\cos x$
      i. Begin by graphing $y = \cos x$.
      ii. Multiplying our y values by 2, we obtain $y = 2\cos x$. By multiplying by 2, our maximum y value is now 2 and our minimum y value is now -2.
      iii. We reflect our graph across the x-axis to produce $y = -2\cos x$. So at $x = 0$, we now have $y = -2$ instead of 2.
      iv. $y = 3 - 2\cos x$ is the same as $y = -2\cos x + 3$, so we move our graph 3 units upwards.

An explicit example to illustrate what was shown in **Test 1 2016 Question 2b**:

$y = 4(-x-3)^3 = 4(-(x+3))^3$ has a horizontal translation of 3 units **left**.

$y = 4(3-x)^3 = 4(-x+3)^3 = 4(-(x-3))^3$ has a horizontal translation of 3 units **right**.

2. The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the linear function $F = \frac{9}{5}C + 32$.

   (a) Sketch the graph of the function.

   - $\frac{9}{5}$ is almost 2. We want to draw a line with slope slightly less than 2.
   - Our function has a y-intercept of 32. In other words, we pass through the point $(0, 32)$.

   (b) What is the slope of the graph? What does it represent?

   - The slope of the graph is $\frac{9}{5}$.
   - This means that for each unit change in °C, we observe a change of $\frac{9}{5}$ °F.

   (c) What does the intercept represent?

   - The y-intercept tells us that 0°C equals 32°F (from plugging in 0 into our equation).

3. Let $f(x) = \sqrt{3-x}$ and $g(x) = 2x - 5$. Compute the following and state the domain.

   (a) $f + g = \sqrt{3-x} + 2x - 5$.

   The $2x-5$ does not impose any restrictions on our domain. The only place that gives us problems is when $3 - x$ is negative. Therefore want to set $3 - x \geq 0$. Rearranging, we obtain that $x \leq 3$. Our domain is:

$$\mathcal{D} = \{x \in \mathbb{R} \mid x \leq 3\}$$

   (b) $f - g = \sqrt{3-x} - (2x - 5) = \sqrt{3-x} - 2x + 5$.

   Once again, the terms outside of the square root do not impose any restrictions on our domain. The only place that gives us problems is when $3 - x < 0$. We end up with the same domain as (a).

   (c) $fg = \left(\sqrt{3-x}\right)(2x - 5)$.

   We don't need to expand here because those terms will just end up outside our square root which will not affect our domain. Again, the only part that we are concerned with is the inner term of the square root. As above, we obtain the exact same domain.

   (d) $f/g = \dfrac{\sqrt{3-x}}{2x-5}$.

   First, we want to make sure that the denominator is non-zero. The denominator equals 0 when $x = \frac{5}{2}$. So $x \neq \frac{5}{2}$ is one condition to include in our domain. We are fine with a numerator that is zero, but we want to make sure that we are not taking the square root of a negative quantity. So once again, we want $3 - x \geq 0 \Longrightarrow x \leq 3$. Our domain is:

$$\mathcal{D} = \left\{ x \in \mathbb{R} \,\middle|\, x \leq 3, x \neq \frac{5}{2} \right\}$$

   (e) $f \circ g = \sqrt{3 - (2x - 5)} = \sqrt{-2x + 8}$.

   We want $-2x + 8 \geq 0 \Longrightarrow 8 \geq 2x \Longrightarrow 4 \geq x \Longrightarrow x \leq 4$. Our domain becomes:

$$\mathcal{D} = \{x \in \mathbb{R} \mid x \leq 4\}$$

(f) $g \circ f = 2\sqrt{3-x} - 5$.

Once again, we want $3 - x \geq 0$. This results in the same domain as (a).

4. Determine whether the following are true or false. Explain your reasoning.

(a) If f and g are linear functions, then $f \circ g$ is a linear function.

This is true! Suppose we have linear functions $f = ax + b$ and $g = cx + d$.

$$\begin{aligned} f \circ g &= a(cx + d) + b \\ &= acx + ad + b \end{aligned}$$

Our result is a linear function with slope ac and y-intercept $ad + b$.

(b) The graph of $y = 2^{-x}$ is the same as the graph of $y = 0.5^x$.

This is true! By properties of exponents:

$$2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x = 0.5^x$$

(c) Since $e < 3$, $e^x < 3^x$ for all x.

This is not true for **<u>all</u>** x.

$$e^2 < 3^2 \implies \frac{1}{e^2} > \frac{1}{3^2} \implies e^{-2} > 3^{-2}$$

Through a counterexample, we have shown that $e^x \not< 3^x$ for **<u>all</u>** x.

(d) The range of $1 - 5\cos(1-x)$ is $-4 \leq y \leq 6$. True!

- We don't actually care about the transformation inside the cos function because that doesn't do anything to our range
- Multiplying by 5 on the outside changes our range from $-1 \leq y \leq 1$ to $-5 \leq y \leq 5$.
- Multiplying by a -1 on the outside makes our maximums become minimums and minimums become maximums, but since we are still hitting these maximums and minimums our range is unchanged.
- The $+1$ on the outside means that we shift vertically up one unit. This changes our range from $-5 \leq y \leq 5$ to $-4 \leq y \leq 6$.

5. Let f be a one-to-one function with domain A and range B.

(a) What is the domain of f^{-1}? What is the range of f^{-1}?

f is a function that maps from A to B. Then its inverse would map from B to A. So the domain of f^{-1} is B and the range of f^{-1} is A.

(b) Suppose $f(x) = x^5 + x^3 + x$. Find $f^{-1}(3)$ and $f(f^{-1}(2))$.

Remember that f^{-1} is a function that takes in y values and outputs x values! So $f^{-1}(3)$ is asking "what x value corresponds to a y value of 3?" (Notice here that it is important f is a one-to-one function!)

$x^5 + x^3 + x = 3$. The only solution is $x = 1$. So $f^{-1}(3) = 1$.

$f^{-1}(2)$ is looking for a x value that corresponds to a y value of 2. Suppose the x value that corresponds to $y = 2$ is $\heartsuit$. Then plugging in $x = \heartsuit$ back into f should give us the corresponding y value, which is 2.

$$f(f^{-1}(2)) = f(\heartsuit) = 2$$

It is also true that $f^{-1}(f(\heartsuit)) = \heartsuit$.