Tutorial 4

Week of February 4, 2019

Question 6.1.12, Page 263

Suppose a certain type of fertilizer has an expected yield per acre of μ_1 with variance σ^2, whereas the expected yield for a second type of fertilizer is μ_2 with the same variance σ^2. Let S_1^2 and S_2^2 denote the sample variances of yields based on sample sizes n_1 and n_2, respectively, of the two fertilizers. Show that the pooled estimator:

$$\hat{\sigma}_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

is an unbiased estimator of σ^2.

Question 6.1.13 (Modified), Page 263

Consider a random sample with probability density function:

$$f(x;\theta) = \frac{1}{2}(1 + \theta x), \quad x \in [-1, 1], \quad \theta \in [-1, 1]$$

For what value of c would $\hat{\theta} = c\overline{X}$ be an unbiased estimator of θ?

Method of Moments

Consider a random sample that is normally distributed with mean μ and variance σ^2. Using the method of moments, obtain point estimates for μ and σ^2.

Maximum Likelihood Estimation

Suppose that the number of Legionella bacteria in a 1 litre sample of water follows a Poisson distribution with unknown parameter λ. Given a random sample $X_1, X_2, \ldots, X_n$:

(a) Derive the MLE of λ. Is it biased or unbiased?

(b) Suppose we are given the following observations:

232 225 249 233 242 203 223 229 224 230 235 217 217 192

Calculate the maximum likelihood estimate for λ.