Tutorial 5

Week of February 11, 2019

Question 6.2.20 (Modified), Page 273

A diagnostic test for a certain disease is applied to n individuals **known to not have the disease**. Let p be the probability that a disease free individual's test result is positive (i.e. p is the true proportion of test results from disease free individuals that are positive).

(a) i. Suppose that of 20 subjects, results of subjects 3, 14, and 19 came back positive. Find and calculate the MLE of p treating this as a series of Bernoulli trials.

ii. Suppose that the exact sequence of test results was unknown but that 3 of the 20 test results came back positive. Find and calculate the MLE of p treating this as a Binomial experiment.

(b) Is the estimator of part (a) unbiased? What is the variance of this estimator? What happens to the variance as $n \to \infty$?

(c) If $n = 20$ and $x = 3$, what is the MLE of the probability $(1-p)^5$ that none of the next five tests done on disease free individuals are positive?

Question 7.1.2, Page 284

Each of the following is a confidence interval for μ, the true mean resonance frequency (Hz) for all tennis rackets of a certain type:

$$(114.4, 115.6) \quad (114.1, 115.9)$$

(a) What is the value of the sample mean resonance frequency?

(b) Both intervals were calculated from the same sample data. The confidence level for one of these intervals is 90% and for the other is 99%. Which is which?

Question 7.1.10, Page 285

A random sample of $n = 15$ heat pumps of a certain type yielded the following observations on lifetime (in years):

2.0	1.3	6.0	1.9	5.1	0.4	1.0	5.3
15.7	0.7	4.8	0.9	12.2	5.3	0.6	

(a) Assume that the lifetime distribution is exponential (mean $= \lambda$) and use the fact that

$$\frac{2n\overline{X}}{\lambda} \sim \chi^2_{2n}$$

to obtain a 95% CI for expected (true average) lifetime.

(b) How should the interval of part (a) be altered to achieve a confidence level of 99%?

(c) What is a 95% CI for the standard deviation of the lifetime distribution? [Hint: What is the standard deviation of an exponential random variable?]